

### 1.3 – Midpoint & Segment Bisector

Objective: I can find the coordinates of the midpoint of a segment.

MIDPOINT of a segment: The point that bisects, or divides, a segment into 2 congruent ( $\cong$ ) segments.

A point is the midpoint of segment if it is between the two endpoints,

and the distances from this point to each endpoint are equal.

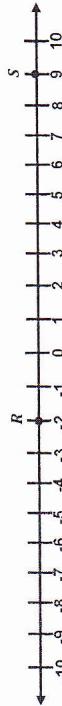
$$\text{* Tick marks indicate } \cong \text{ segments!} \quad \overline{PM} \cong \overline{MQ} \quad PM = MQ$$

Midpoint on a number line:

$$\text{Midpoint} = \frac{a+b}{2}$$

(Where  $a$  &  $b$  are coordinates of endpoints.)

EXAMPLE 1: Find the coordinate of the midpoint of  $\overline{RS}$ .



$$M = \frac{-2+9}{2} = \frac{7}{2} = 3.5$$

EXAMPLE 2:  $B$  is the midpoint of  $\overline{AC}$ .  $AB = z + 2$  and  $BC = 2z - 6$ . Find "z".

$$\begin{aligned} z+2 &= 2z-6 \\ 8 &= z \end{aligned}$$

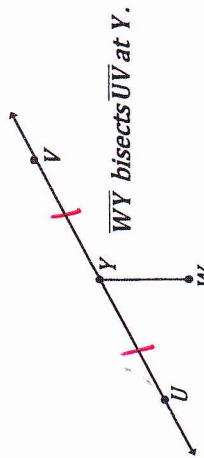
**SEGMENT BISECTOR:** A ray, segment, or line that intersects a segment at its midpoint.

EXAMPLE 1:  $B$  is between  $A$  and  $C$ .  $AB = 2y + 6$ ,  $BC = y + 8$ , and  $AC = 20$ . Find the value of "y" and determine if  $B$  is a bisector.

$$\begin{aligned} AB &= 2(y+6) & AC &= 20 \\ BC &= y+8 & 2y+12+y+8 &= 20 \\ 3y+20 &= 20 & 3y &= 0 \\ 3y &= 6 & y &= 2 \end{aligned}$$

Since  $AB = BC$ ,  $B$  is the midpoint of  $\overline{AC}$ .

Use the figure below to answer EXAMPLES 2 – 5.



EXAMPLE 2: If  $UY = 5$ , then find  $YV$  and  $UV$ .

$$\begin{aligned} YV &= 5 \\ UV &= 10 \end{aligned}$$

EXAMPLE 4: If  $UV = 18$  and  $UY = 9$ , find  $YV$ .

$$\begin{aligned} YV &= 9 \\ UV &= 18 \\ 9+9 &= 18 \\ 18 &= 18 \end{aligned}$$

$$YV = 8 - 1 = 7$$

EXAMPLE 3: If  $UY = 4x - 3$  and  $YV = x - 1$ , find  $UV$ .

$$\begin{aligned} 4x-3 &= x \\ -3 &= -3x \\ 1 &= x \end{aligned}$$

$$\begin{aligned} UV &= 1 \\ UV &= 2 \\ UV &= 2 \end{aligned}$$

Midpoint on a coordinate plane:

$$M(x,y) = \left( \frac{x_1+x_2}{2}, \frac{y_1+y_2}{2} \right)$$

Endpoints  
 $(x_1, y_1)$   
 $(x_2, y_2)$

EXAMPLE 1: Find the midpoint between  $(-11, 3)$  and  $(8, -7)$ .

$$x_1 \quad y_1 \quad x_2 \quad y_2$$

$$M(x,y) = \left( \frac{-11+8}{2}, \frac{3-7}{2} \right) = \left( \frac{-3}{2}, \frac{-4}{2} \right) = \left( \frac{-3}{2}, -2 \right)$$

EXAMPLE 2: Find the coordinates of the midpoint of  $\overline{VW}$ , if

$$V(3, -6) \text{ and } W(7, 2).$$

$x_1 \quad y_1 \quad x_2 \quad y_2$

$$M(x,y) = \left( \frac{3+7}{2}, \frac{-6+2}{2} \right) = \left( \frac{10}{2}, \frac{-4}{2} \right) = (5, -2)$$

EXAMPLE 3:  $M$  is the midpoint of  $\overline{AB}$  with  $A(0, 1)$  and  $M(3, 5)$ .

Find the coordinates of  $B$ .

$$(0, 1) \quad (3, 5) \quad (x_2, y_2)$$

$x_1 \quad y_1 \quad x_2 \quad y_2$

$$(3, 5) = \left( \frac{0+x_2}{2}, \frac{1+y_2}{2} \right)$$
$$3 = \frac{x_2}{2}$$
$$5 = \frac{1+y_2}{2}$$
$$6 = x_2$$
$$10 = 1 + y_2$$
$$9 = y_2$$

$B(6, 9)$

EXAMPLE 4: The midpoint of  $\overline{RQ}$  is  $M(4, -1)$ . What are the coordinates of  $R$  if  $Q$  is at  $(3, -2)$ ?

$$(x_1, y_1) \quad (4, -1) \quad (3, -2)$$

$x_1 \quad y_1 \quad x_2 \quad y_2$

$$(4, -1) = \left( \frac{x_1+3}{2}, \frac{y_1-2}{2} \right)$$
$$4 = \frac{x_1+3}{2}$$
$$-1 = \frac{y_1-2}{2}$$
$$8 = x_1 + 3$$
$$-2 = y_1 - 2$$
$$5 = x_1$$
$$0 = y_1$$

$$R(5, 0)$$