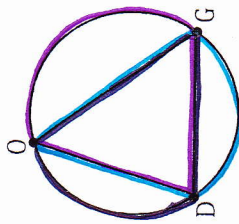


NOTES 10.4 – INSCRIBED ANGLES

INSCRIBED ANGLE:

An angle whose vertex is on the circle and whose sides are chords of the circle

EXAMPLE 1: Name ALL of the inscribed angles and their corresponding intercepted arcs below.

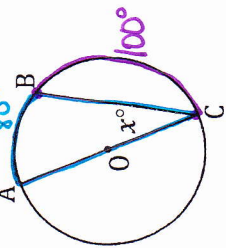


Inscribed angles/Intercepted Arc:

- $\angle DOG / \widehat{DG}$
- $\angle OGD / \widehat{DO}$
- $\angle GDO / \widehat{GO}$

THEOREM: If an angle is inscribed in a circle, then the measure of the angle is half the measure of the intercepted arc.

EXAMPLE 2: Given that $m\widehat{BC} = 100^\circ$, find the value of 'x' in circle O.



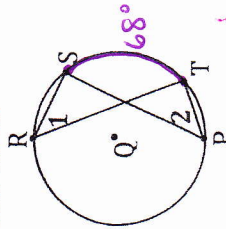
$$m\angle ACB = \frac{1}{2} m\widehat{AB}$$

$$x = \frac{1}{2}(80)$$

x = 40

THEOREM: If two inscribed angles of a circle or congruent circles intercept congruent arcs or the same arc, then the angles are congruent.

EXAMPLE 3: In circle Q, $m\widehat{ST} = 68^\circ$. Find the $m\angle 1$ and $m\angle 2$.



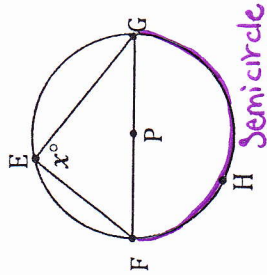
$$m\angle 1 = \frac{1}{2} m\widehat{ST}$$

$$m\angle 2 = \frac{1}{2} m\widehat{ST}$$

$m\angle 1 =$ 34° $m\angle 2 =$ 34°

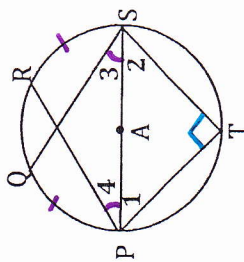
THEOREM: If an inscribed angle of a circle intercepts a semicircle, then the angle is a right angle.

EXAMPLE 4: Find the value of 'x'.



x = 90

EXAMPLE 5: In circle A, $m\angle 1 = (6x + 11)^\circ$, $m\angle 2 = (9x + 19)^\circ$, $m\angle 3 = (4y - 25)^\circ$, $m\angle 4 = (3y - 9)^\circ$, and $\widehat{PQ} \cong \widehat{RS}$. Find $m\angle 1$, $m\angle 2$, $m\angle 3$, and $m\angle 4$.



$$m\angle 4 = m\angle 3$$

$$3y - 9 = 4y - 25$$

$$16 = y$$

$$m\angle 3 = 4(16) - 25 = 39^\circ$$

$$m\angle 4 = 3(16) - 9 = 39^\circ$$

$$m\angle 1 + m\angle 2 = 90$$

$$6x + 11 + 9x + 19 = 90$$

$$15x + 30 = 90$$

$$15x = 60$$

$$x = 4$$

$$m\angle 1 = 6(4) + 11 = 35^\circ$$

$$m\angle 2 = 9(4) + 19 = 55^\circ$$

$m\angle 1 =$ 35°

$m\angle 2 =$ 55°

$m\angle 3 =$ 39°

$m\angle 4 =$ 39°

THEOREM: If a quadrilateral is inscribed in a circle, then its opposite angles are supplementary.

EXAMPLE 6: Quadrilateral QRST is inscribed in circle C. If

$m\angle T = 95^\circ$, $m\angle S = 100^\circ$, $\widehat{TR} = 160^\circ$, and $\widehat{QS} = 170^\circ$, find $m\angle Q$ and $m\angle R$.

$m\angle Q = \frac{1}{2}(160)$

$m\angle Q = 80^\circ$

$m\angle R = \frac{1}{2}(170)$

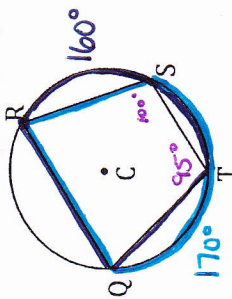
$m\angle R = 85^\circ$

$m\angle Q + m\angle S = 180$

$80 + 100 = 180$

$m\angle R + m\angle T = 180$

$85 + 95 = 180$

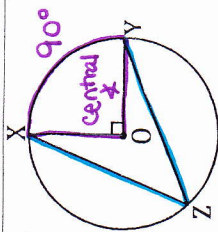


EXAMPLE 7: Find the value of the inscribed angle.

$m\angle XZY = \frac{1}{2} m\widehat{XY}$

$m\angle XZY = \frac{1}{2}(90)$

$m\angle XZY = 45^\circ$



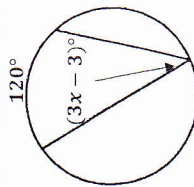
EXAMPLE 8: Find the value of 'x'.

$3x - 3 = \frac{1}{2}(120)$

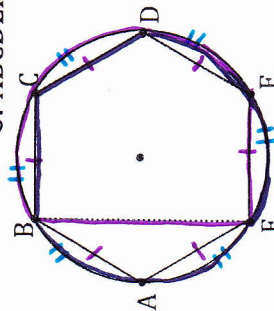
$3x - 3 = 60$

$3x = 63$

$x = 21$



EXAMPLE 9: Hexagon ABCDEF is inscribed in circle O. All sides of ABCDEF are congruent. Find the following.



Each arc is $\frac{1}{6}$ of the circle.

a) $m\widehat{CD} = 60^\circ$

$m\angle BFE = \frac{1}{2}(180)$

$m\angle BFE = 90^\circ$

$m\angle BCD = \frac{1}{2}(240)$

$m\angle BCD = 120^\circ$