NOTES 2.2: Biconditionals & Reasoning

Objective: I can write biconditional statements & use reasoning to form conjectures.

Biconditional Statement:

A statement that can be written in the form "p if and only if q". This means "if p, then q" and "if q, then p." * iff = if and only if

Write the conditional statement and converse within the biconditional.

Example 1: An angle is obtuse if and only if its measure is greater than 90° and less than 180°.

Conditional	If an ∠ is obtuse, then its measure is greater than 90° and less than 180°.
Converse	In an ∠, is measure is greater than 90° and less than 180°, then it is obtuse.

Example 2: A solution is neutral iff its pH is 7.

Conditional	If a solution is neutral, then its pH is 7.
Converse	If a solution's pH is 7, then it is neutral.

For each conditional, write the converse and a biconditional statement.

Example 3: If $5x - 8 = 37$, then $x = 9$.

Converse	If $x = 9$, then $5x - 8 = 37$.
Biconditional	$5x - 8 = 37$ iff $x = 9$.

Example 4: If two angles have the same measure, then they are congruent.

Converse	If two angles are congruent, then they have the same measure.
Biconditional	Two angles have the same measure iff they are congruent.

For a biconditional statement to be true, both the conditional statement and its converse must be true. If either the conditional or the converse is false, then the biconditional statement is false.

Determine if the biconditional is true. If false, give a counterexample.

Example 5: A rectangle has side lengths of 12 cm and 25 cm if and only if its area is 300 cm^2.

	True / False

Counterexample: Side lengths of 10cm & 30cm

Example 6: An angle is a right angle iff its measure is 90°.

	True / False

Counterexample: None

Write each definition as a biconditional.

Example 7: A pentagon is a five-sided polygon.

A figure is a pentagon iff it is a 5-sided polygon.

Example 8: A right angle measures 90°.

An angle is a right angle iff it measures 90°.

Inductive Reasoning: The process of reasoning that a rule of statement is true because specific cases are true.
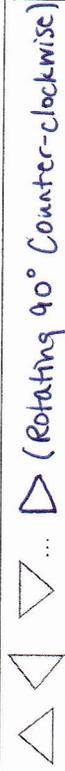
Conjecture: A conclusion based on a pattern.

Find the next item in each pattern.

Example 9: January, March, May, ... July (Alternating months)

Example 10: 7, 14, 21, 28, ... 35 (Multiples of 7)

Example 11: △ ◁ ▽ ... ▷ (Rotating 90° Counter-clockwise)

Complete each conjecture.

Example 12: The sum of two positive numbers is ___positive___.

Example 13: The number of lines formed by 4 points, no three of which are collinear, is ___6___.

*To show that a conjecture is always true, you must prove it.
*To show that a conjecture is false, you have to find only one example in which the conjecture is not true.

Counterexample:

One example in which a conjecture is not true.

A counterexample can be a drawing, a statement, or a number.

Inductive Reasoning

1. Look for a pattern.
2. Make a conjecture.
3. Prove the conjecture or find a counterexample.

Show that each conjecture is false by finding a counterexample.

Example 14: For every integer n, n^3 is positive.

Counterexample: Try $n=1$. $1^3=1$, which is positive. Try $n=-3$. $-3^3 = -27$, which is negative. $n=-3$ is a counterexample.

Example 15: Two complementary angles are not congruent.

Counterexample: If 2 angles both measure $45°$, then they are $\cong$. They also add to $90°$, so they are complementary.

Example 16: The monthly high temperature in Abilene is never below $90°F$ for two months in a row.

Monthly High Temperatures (°F) in Abilene, Texas

Jan	Feb	Mar	Apr	May	Jun	Jul	Aug	Sep	Oct	Nov	Dec
88	89	97	99	107	109	110	107	106	103	92	89

Counterexample: Since the temperatures in January and February were below 90°, the conjecture is false.

Determine if each conjecture is true. If false, give a counterexample.

Example 17: The quotient of two negative numbers is a positive number. True / False

Counterexample: None

Example 18: Two supplementary angles are not congruent. True / False

Counterexample: Two 90° angles are supplementary and $\cong$.

Deductive Reasoning:

The process of using logic to draw conclusions from given facts, definitions, & properties.

State whether each conclusion uses inductive or deductive reasoning.

Example 19: There is a myth that you can balance an egg on its end only on the spring equinox. A person was able to balance an egg on July 8, September 21, and December 19. Therefore, this myth is false.

Inductive – Based on a pattern

Example 20: There is a myth that the Great Wall of China is the only man-made object visible from the Moon. The Great Wall is barely visible in photographs taken from 180 miles above Earth. The Moon is about 237,000 miles from Earth. Therefore, the myth cannot be true.

Deductive – Based on facts

Example 21: Opposite rays are two rays that have a common endpoint and form a line. $\overrightarrow{YX}$ and $\overrightarrow{YZ}$ are opposite rays.

Deductive – Based on a definition