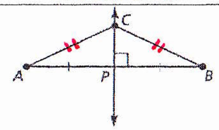
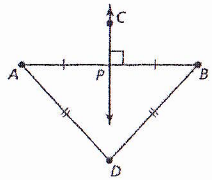
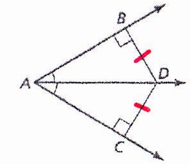
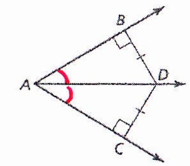
NOTES: 6.1 – Perpendicular and Angle Bisectors

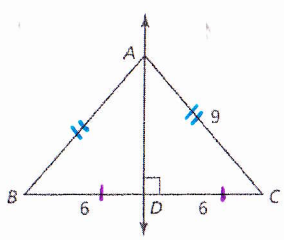
Objective: I can use properties of Δs to find ∡ measures & side lengths.

Equidistant: the same distance from

THEOREM	DIAGRAM
PERPENDICULAR BISECTOR THEOREM In a plane, if a point lies on the perpendicular bisector of a segment, then it is equidistant from the endpoints of the segment.	 **If** $\overleftrightarrow{CP}$ **is the** $\perp$ **bisector of** $\overline{AB}$**, then** $\overline{CA} \cong \overline{CB}$.
CONVERSE OF THE PERPENDICULAR BISECTOR THEOREM In a plane, if a point is equidistant from the endpoints of a segment, then it lies on the perpendicular bisector of the segment.	 **If** $DA = DB$**, then point** D **lies on the** $\perp$ **bisector of** $\overline{AB}$.

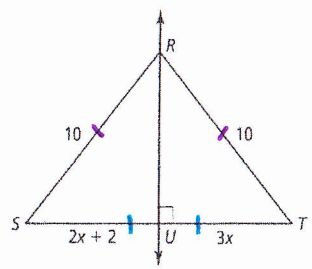
THEOREM	DIAGRAM
ANGLE BISECTOR THEOREM If a point lies on the bisector of an angle, then it is equidistant from the two sides of the angle.	 **If** $\overrightarrow{AD}$ **bisects** $\angle BAC$ **and** $\overline{DB} \perp \overrightarrow{AB}$ **and** $\overline{DC} \perp \overrightarrow{AC}$**, then** $\overline{BD} \cong \overline{DC}$.
CONVERSE OF THE ANGLE BISECTOR THEOREM If a point is in the interior of an angle and is equidistant from the two sides of the angle, then it lies on the bisector of the angle.	 **If** $\overline{DB} \perp \overrightarrow{AB}$ **and** $\overline{DC} \perp \overrightarrow{AC}$ **and** $DB = DC$**, then** $\overrightarrow{AD}$ **bisects** ∡BAC.

Example 1:
Find AB and explain your reasoning.



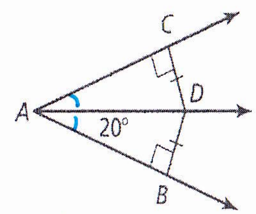
Since BD = DC & ∡ADC is a rt ∡, $\overleftrightarrow{AD}$ is the ⊥ bisector of $\overline{BC}$.
So, $\overline{AC} \cong \overline{AB}$.
Therefore, AB = 9.

Example 2:
Find SU and explain your reasoning.



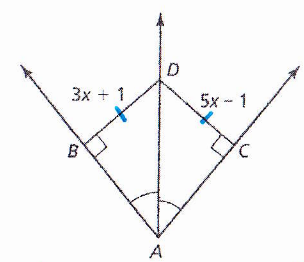
Since RS = RT, R is a point on the ⊥ bisector of $\overline{ST}$.
So, $\overline{SU} \cong \overline{UT}$.
Therefore, $2x+2=3x$
$2=x$.
$SU=2(2)+2=6$

Example 3:
Find $m\angle CAB$ and explain your reasoning.



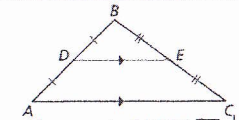
Since D is in the interior of ∡CAB, $\overline{CD} \perp \overrightarrow{AC}$, $\overline{DB} \perp \overrightarrow{AB}$, & $\overline{CD} \cong \overline{DB}$, D lies on the bisector of ∡CAB.
So, ∡CAD ≅ ∡DAB.
Therefore, m∡CAD = 20° & m∡CAB = 40°.

Example 4:
Find BD and explain your reasoning.



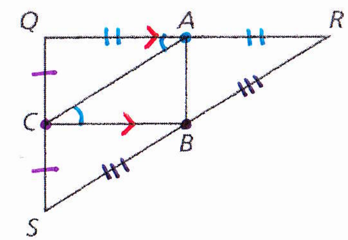
Since D is a point on the ∡ bisector of ∡BAC, $\overline{DB} \perp \overrightarrow{AB}$, & $\overline{DC} \perp \overrightarrow{AC}$, $\overline{DB} \cong \overline{DC}$.
So, $3x+1=5x-1$
$-2x=-2$
$x=1$.
Therefore, $BD=3(1)+1$
$=4$.

NOTES 6.4 – TRIANGLE MIDSEGMENT THEOREM

THEOREM	DIAGRAM
TRIANGLE MIDSEGMENT THEOREM The segment connecting the midpoints of two sides of a triangle is parallel to the third side and is half as long as that side.	 $\overline{DE}$ is a midsegment of ΔABC, $\overline{DE} \parallel \overline{AC}$, and DE = $\frac{1}{2}AC$.

EXAMPLES:

In Examples 1–6, use ΔQRS where *A*, *B*, and *C* are the midpoints of the sides.



1. **When** $AB = 16$, **what is** QS?

   $AB = \frac{1}{2}(QS)$
   $16 = \frac{1}{2}(QS)$
   $32 = QS$

2. **When** $SR = 68$, **what is** CA?

   $CA = \frac{1}{2}(SR)$
   $CA = \frac{1}{2}(68)$
   $CA = 34$

3. **When** $SR = 46$, **what is** BR?

   $BR = \frac{1}{2}(SR)$
   $BR = \frac{1}{2}(46)$
   $BR = 23$

4. **When** $CA = 3x - 1$ and $SR = 5x + 4$, **what is** CA?

   $CA = \frac{1}{2}(SR)$
   $3x - 1 = \frac{1}{2}(5x+4)$
   $3x - 1 = \frac{5}{2}x + 2$
   $\frac{1}{2}x = 3$
   $x = 6$
   $CA = 3(6) - 1$
   $CA = 18 - 1$
   $CA = 17$

5. **When** $QS = 6x$ and $CS = 5x - 8$, **what is** AB?

   $CS = \frac{1}{2}(QS)$
   $5x - 8 = \frac{1}{2}(6x)$
   $5x - 8 = 3x$
   $2x = 8$
   $x = 4$
   $QS = 6x$
   $QS = 6(4)$
   $QS = 24$
   $AB = \frac{1}{2}(QS)$
   $AB = \frac{1}{2}(24)$
   $AB = 12$

6. **When** $m\angle BCA = 48°$, **what is the** $m\angle CAQ$?

   $m\angle CAQ = 48°$