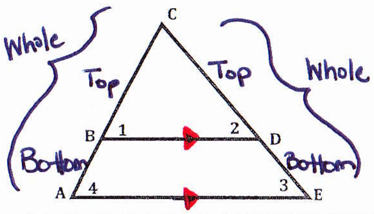
NOTES 8.4: PARALLEL LINES & PROPORTIONAL PARTS

Objective: I can use theorems to solve problems involving // lines & proportional parts.

Proportions can be used to find the lengths of segments determined by parallel lines.

TRIANGLE PROPORTIONALITY THEOREM: If a line is parallel to one side of a triangle and intersects the other two sides in two distinct points, then it separates these sides into segments of proportional length.



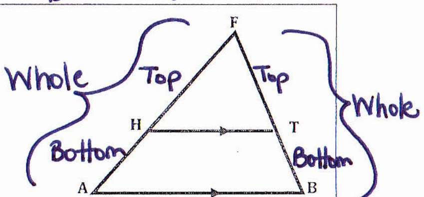
$\frac{CB}{CA} = \frac{CD}{CE}$ (T/W = T/W) $\quad \frac{CB}{CD} = \frac{CA}{CE}$ (T/T = W/W) $\quad \frac{BA}{CA} = \frac{DE}{CE}$ (B/W = B/W)

$\frac{CB}{BA} = \frac{CD}{DE}$ (T/B = T/B) $\quad \frac{CB}{CD} = \frac{BA}{DE}$ (T/T = B/B) $\quad \frac{BA}{DE} = \frac{CA}{CE}$ (B/B = W/W)

EXAMPLE 1: TRUE or FALSE?

(a) $\frac{FA}{HA} = \frac{FB}{TB}$ T $\qquad$ (b) $\frac{FT}{FH} = \frac{FB}{FA}$ T

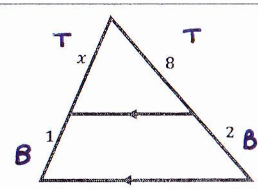
(c) $\frac{FH}{FT} = \frac{HA}{TB}$ T $\qquad$ (d) $\frac{FA}{FH} = \frac{FT}{TB}$ F



EXAMPLE 2: Find the value of 'x'.

$\frac{T}{B} = \frac{T}{B}$ $\qquad \frac{x}{1} = \frac{8}{2}$

$x = 4$



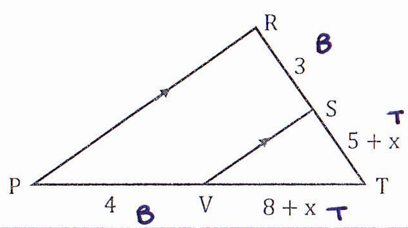
EXAMPLE 3: Find the value of 'x'.

$\frac{T}{B} = \frac{T}{B}$ $\qquad \frac{(5+x)}{3} = \frac{(8+x)}{4}$

$4(5+x) = 3(8+x)$

$20 + 4x = 24 + 3x$

$x = 4$



Likewise, proportional parts of a triangle can be used to prove the converse of this theorem.

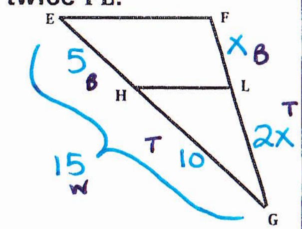
THEOREM: If a line intersects two sides of a triangle and separates the sides into corresponding segments of proportional lengths, then the line is parallel to the third side.

EXAMPLE 4: In $\triangle EFG$, $EG = 15$, $EH = 5$, and LG is twice FL. Determine whether $\overline{HL} \parallel \overline{EF}$.

$\frac{T}{B} = \frac{T}{B}$ $\qquad \frac{10}{5} = \frac{2x}{x}$ $\qquad$ Cross Multiply!

$10x = 10x$ ✓

Yes!



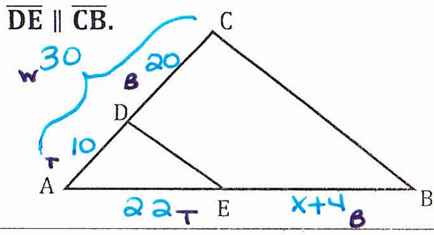
EXAMPLE 5: In $\triangle ABC$, find 'x' so that $\overline{DE} \parallel \overline{CB}$.

$AC = 30 \qquad AD = 10$

$AE = 22 \qquad EB = x + 4$

$\frac{T}{B} = \frac{T}{B}$ $\qquad \frac{10}{20} = \frac{22}{x+4}$ $\qquad x + 4 = 44$

$x = 40$



THEOREM: A segment whose endpoints are the midpoints of two sides of a triangle is parallel to the third side of the triangle, and its length is half the length of the third side.

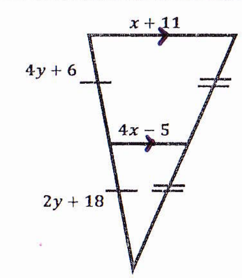
EXAMPLE 6: Find the values of 'x' and 'y'.

$2(4x-5) = x + 11 \qquad 4y + 6 = 2y + 18$

$8x - 10 = x + 11 \qquad 2y = 12$

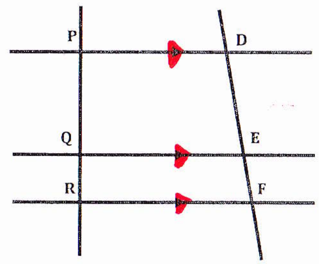
$7x = 21 \qquad y = 6$

$x = 3$



THEOREM: If three or more parallel lines intersect two transversals, then they cut off the transversals proportionally.

$\frac{PQ}{QR} = \frac{DE}{EF}$ OR $\frac{PQ}{DE} = \frac{QR}{EF}$



* These are the only 2 ways to set these up!

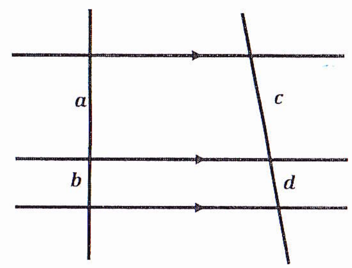
EXAMPLE 7: TRUE or FALSE?

(a) $\frac{a}{b} = \frac{c}{d}$ T
$ad = bc$

(b) $\frac{a}{c} = \frac{c}{d}$ F
$ad = c^2$

(c) $\frac{a}{d} = \frac{c}{b}$ F
$ab = dc$

(d) $\frac{b}{c} = \frac{a}{d}$ F
$bd = ac$



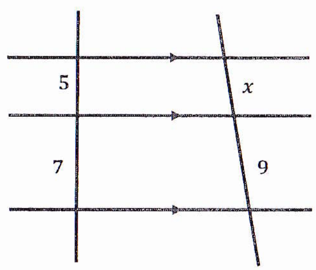
EXAMPLE 8: Find the value of 'x'.

Only 2 ways to set up!

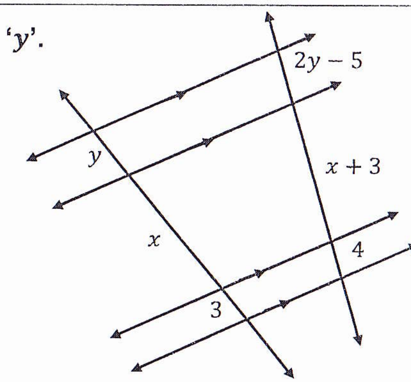
$\frac{5}{7} = \frac{x}{9}$
$7x = 45$
$x = \frac{45}{7}$

OR

$\frac{5}{x} = \frac{7}{9}$
$7x = 45$
$x = \frac{45}{7}$



EXAMPLE 9: Find the values of 'x' and 'y'.

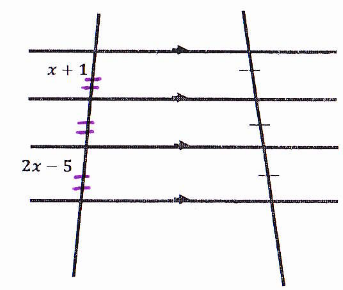


$\frac{y}{3} = \frac{2y-5}{4}$
$4y = 6y - 15$
$-2y = -15$
$y = \frac{15}{2}$

$\frac{x}{3} = \frac{x+3}{4}$
$4x = 3x + 9$
$x = 9$

THEOREM: If three or more parallel lines cut off congruent segments on one transversal, then they cut off congruent segment on every transversal.

EXAMPLE 10: Find the value of 'x'.



$x + 1 = 2x - 5$
$6 = x$