12.4 – Quadratic Equations with Complex Solutions

If $ax^2 + bx + c = 0$, then $x = \frac{-b \pm \sqrt{b^2-4ac}}{2a}$.

The discriminant of a quadratic equation is represented by $b^2 - 4ac$.

The discriminant is used to find the number of solutions the quadratic equation has. If the value of the discriminant is 0, the equation has 1 solution. If the value of the discriminant is any other number, the equation has 2 solutions.

The discriminant is also used to tell us what type of solutions the quadratic equation has.

Value of the Discriminant	Number & Type of Solutions
$b^2 - 4ac > 0$ (Positive number, not 0.)	2 Real Solutions
$b^2 - 4ac = 0$ (Exactly 0.)	1 Real Solution
$b^2 - 4ac < 0$ (Negative number, not 0.)	2 Imaginary Solutions

Find the number and type of solutions for the following quadratic equations using the discriminant. Then, solve each equation using the quadratic formula.

1. $3x^2 - 5x - 2 = 0$

b^2-4ac
$(-5)^2-4(3)(-2)$
$25+24$
49

$x = \frac{-b \pm \sqrt{b^2-4ac}}{2a}$
$x = \frac{5 \pm \sqrt{49}}{2(3)}$
$x = \frac{5 \pm 7}{6}$
$x = \frac{5+7}{6}$ $x = \frac{5-7}{6}$

$a =$ 3 $b =$ -5 $c =$ -2

Number & Type of Solutions: 2 Real

Solution: $x = 2, \frac{-1}{3}$

2. $x^2 - 6x + 9 = 0$

b^2-4ac
$(-6)^2-4(1)(9)$
$36-36$
0

$x = \frac{-b \pm \sqrt{b^2-4ac}}{2a}$
$x = \frac{6 \pm \sqrt{0}}{2(1)}$
$x = \frac{6}{2}$

$a =$ 1 $b =$ -6 $c =$ 9

Number & Type of Solutions: 1 Real

Solution: $x = 3$

3. $x^2 - 2x + 5 = 0$

b^2-4ac
$(-2)^2-4(1)(5)$
$4-20$
-16

$x = \frac{-b \pm \sqrt{b^2-4ac}}{2a}$
$x = \frac{2 \pm \sqrt{-16}}{2(1)}$
$x = \frac{2 \pm 4i}{2}$
$x = \frac{2}{2} \pm \frac{4}{2}i$
$x = 1 \pm 2i$

$a =$ 1 $b =$ -2 $c =$ 5

Number & Type of Solutions: 2 Imag.

Solution: $x = 1 \pm 2i$